A22648

Calculators permitted provided they are not capable of being used to store alphabetical information other than hexadecimal numbers.

UNIVERSITY OF BIRMINGHAM

School of Computer Science

MSc Computer Science
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Final Year – MSci Computer Science
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**06 20008**

# Cryptography

Summer May/June Examinations 2017

Time allowed: 1 hour 30 minutes

[Answer ALL Questions]

1. DES is a block cipher with an effective key length of 56 bits (that is, there are $2^{56}$ distinct keys). 3DES is a block cipher defined in terms of DES. 3DES uses three DES keys $k_1$, $k_2$ and $k_3$. The encryption $c$ of a block $m$ using 3DES is given by $c = \mathsf{Enc}_{k_1}(\mathsf{Dec}_{k_2}(\mathsf{Enc}_{k_3}(m)))$.

   (a) Explain why 3DES is better than DES. **[7%]**

   (b) Explain why encryption with 3DES is defined as $\mathsf{Enc}_{k_1}(\mathsf{Dec}_{k_2}(\mathsf{Enc}_{k_3}(m)))$ and not $\mathsf{Enc}_{k_1}(\mathsf{Enc}_{k_2}(\mathsf{Enc}_{k_3}(m)))$. **[7%]**

   Joe is considering to use "2DES", which he defines using

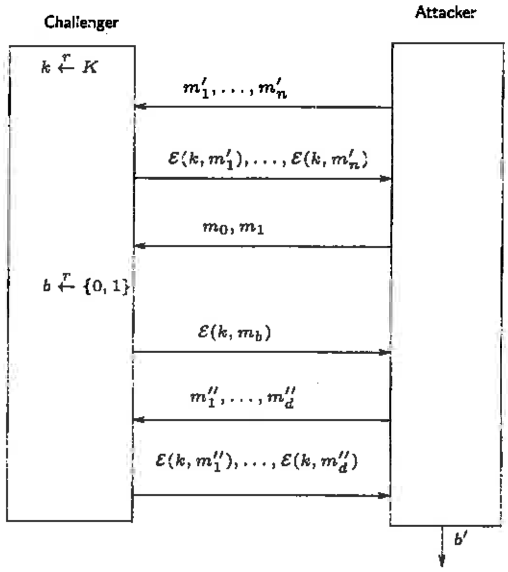
   2DES: $c = \mathsf{Enc}_{k_1}(\mathsf{Enc}_{k_2}(m))$

   (it uses only two DES keys). He hopes that this will give him approximately twice the bitlength security of DES.

   (c) Show that the equation (2DES) above may be equivalently written $\mathsf{Enc}_{k_2}(m) = \mathsf{Dec}_{k_1}(c)$, which has no nested Enc or Dec funcions. **[6%]**

   (d) Suppose an attacker has a plaintext message $m$ and the corresponding ciphertext $c$. Explain how it can find the keys $k_1$ and $k_2$ used by Joe, using a little more than $2^{56}$ operations (and way less than Joe's expected $2^{112}$ operations). **[5%]**

2. The figure below is to remind you of the IND-CPA game.




(a) What is the condition on the output $b'$ that indicates that the attacker has won the game? **[6%]**

For each of the following definitions of $\mathcal{E}(k,m)$ explain whether the attacker can expect to win the game.

(b) $\mathcal{E}(k,m)$ is defined as the encryption of a message $m$ using AES in counter mode, the key $k$, and a randomly-chosen IV. **[7%]**

(c) $\mathcal{E}(k,m)$ is defined as the HMAC of a message $m$ using SHA-256 as the underlying hash function, and the key $k$. **[7%]**

You are designing a system which will encrypt and save some information $m$ on a disk, and later retrieve and decrypt the information. You want to use *authenticated encryption.* You have an encryption function $\mathcal{E}$ satisfying IND-CPA and a MAC function $\mathcal{M}$ satisfying the MAC unforgeability game, and two secret keys $k_1$ and $k_2$.

(d) Which of the following ways to do it is better?

(i) Encrypt-then-MAC: encrypt the message, then compute MAC of ciphertext. The result may be written $\mathcal{E}(k_1,m)$, $\mathcal{M}(k_2,\mathcal{E}(k_1,m))$.

(ii) Encrypt and MAC: The result consists of the encryption of $m$ and the MAC of $m$, and may be written $\mathcal{E}(k_1,m)$, $\mathcal{M}(k_2,m)$.

Explain your answer. **[5%]**

3. Let us consider **RSA-D** = (Kg,Enc,Dec), a variant of the RSA public key encryption scheme.

- Key generation KG($\lambda$)
  - Generate two distinct odd primes $p$ and $q$ of same bit-size $\lambda$
  - Compute $N = p \cdot q$ and $\phi = (p-1)(q-1)$
  - Select two random integers $1 < e_1, e_2 < \phi$ such that $\gcd(e_1, \phi) = 1$ and $\gcd(e_2, \phi) = 1$
  - Compute the unique integer $1 < d_1 < \phi$ such that $e_1 \cdot d_1 \equiv 1 \bmod \phi$
  - Compute the unique integer $1 < d_2 < \phi$ such that $e_2 \cdot d_2 \equiv 1 \bmod \phi$
  - The public key is $PK = (N, e_1, e_2)$. The private key is $SK = (d_1, d_2)$
- Encryption Enc($PK, m$) a message $m \in \mathbf{Z}_N^\star$ proceeds as follows:
  - Generate a random integer $r$ in $\mathbf{Z}_N^\star$
  - Compute $c_1 = r^{e_1} \bmod N$
  - Compute $c_2 = m^{e_2} \cdot r^{e_1 \cdot e_2} \bmod N$
  - Output $C = c_1 || c_2$

(a) Give the corresponding decryption algorithm Dec($SK, C$). Prove your decryption algorithm is correct, i.e. that given a legitimate key pair $(PK, SK)$ it holds Dec($SK$, Enc($PK, m$)) $= m$ for any admissible $m$. **[10%]**

(b) Let us study the security of the asymmetric encryption scheme **RSA-D**:

(i) Describe in technical terms what the statement *"Breaking the RSA problem is hard"* means. **[5%]**

(ii) What is the definition of *one-wayness* for a public key encryption scheme? **[5%]**

(iii) Is **RSA-D** a one-way secure public key encryption scheme? Justify your answer. **[5%]**

(iv) Is **RSA-D** an IND-CPA (or semantically secure) public key encryption scheme? Justify your answer. **[5%]**

4. Let us consider ElGamal encryption parameters $(G, g, p, q)$ for large primes $p, q$, where $g$ is a generator of a subgroup $G$ of $\mathbf{Z}_p^*$ and $G$ has $q$ elements. Recall that in ElGamal every user chooses a random private key $x \in \mathbf{Z}_q$ and computes the public key $X = g^x \bmod p$. To encrypt a message $m \in G$ for a user with public key $X$, the sender chooses a random $y \in \mathbf{Z}_q$ and computes the ciphertext $(g^y, X^y \cdot m)$.

   (a) How is ElGamal encryption related to the Diffie-Hellman key exchange protocol? Describe the Diffie-Hellman (DH) protocol in detail. **[5%]**

   (b) Why is DH key exchange insecure against man-in-the-middle (MitM) attacks? Describe a MitM attack against the DH protocol in detail. **[5%]**

   (c) Present an improvement of the DH key exchange that prevents MitM attacks. Explain your answer. **[10%]**

**Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so**

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- Unauthorised materials (e.g. notes or tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
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- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
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