Theories of Computation: Summative Assignment 3

This exercise is about context free grammars on the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$. You may use the following facts that were mentioned in Week 3.

- Whether a given CFG accepts a given word is decidable.
- Whether the language of a given CFG is empty is decidable.
- Whether the language of a given CFG is Σ^* is undecidable.

Say that a CFG is *red* when it accepts every word of length 3 that begins with $\mathtt{a}$, and *extremely red* when it accepts every word that begins with $\mathtt{a}$.

1. Is redness decidable? Is it semidecidable? Explain your answer. **[3 marks]**
2. Is extreme redness decidable? Is it semidecidable? Explain your answer. (Hint: you may find it helpful to think about the negation of this property.) **[3 marks]**