Theories of Computation: Summative Assignment 1

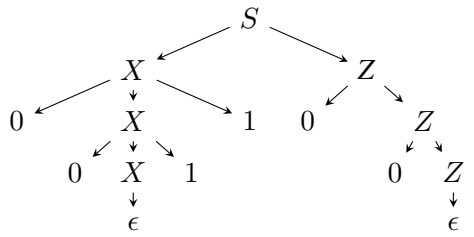
To be handed in on Canvas before **Thursday 3rd March, 5pm GMT**

Exercise 1 *We consider the alphabet* $\Sigma = \{0, 1\}$. *We want to study the following grammar* G*:*

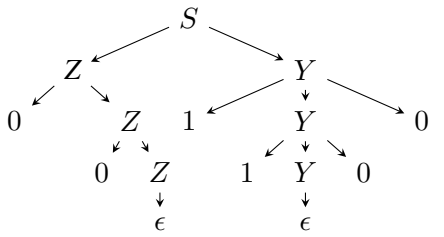
$$\begin{array}{rlcll} \Rightarrow & S & ::= & ZY & \mid & XZ \\ & X & ::= & 0X1 & \mid & \epsilon \\ & Y & ::= & 1Y0 & \mid & \epsilon \\ & Z & ::= & 0Z & \mid & \epsilon \end{array}$$

1. *Give a derivation tree in* G *for the word* 001100 *and the corresponding left-most derivation.* **[2 marks]**
2. *Is the above grammar* G *ambiguous? Justify your answer.* **[2 marks]**
3. *Are the following words in* $L(G)$*? Simply answer yes or no.* **[2 marks]**
   (a) 01000
   (b) 1
   (c) 01100
   (d) ϵ
4. *What is the language* $L(G)$ *generated by this grammar?* **[2 marks]**

Solution 1 *1. In this grammar* G*, there are two possible derivation trees for* 001100 *displayed below with the corresponding left-most derivations.*



$$S \Rightarrow XZ \Rightarrow 0X1Z \Rightarrow 00X11Z \Rightarrow 0011Z \Rightarrow 00110Z \Rightarrow 001100Z \Rightarrow 001100$$



$$S \Rightarrow ZY \Rightarrow 0ZY \Rightarrow 00ZY \Rightarrow 00Y \Rightarrow 001Y0 \Rightarrow 0011Y00 \Rightarrow 001100$$

2. *Because there are two distinct derivation trees for* 001100*,* G *is ambiguous.*
3. *(a) Yes; (b) No; (c) Yes; (d) Yes.*
4. *The grammar*

$$\begin{array}{rlcll} \Rightarrow & S & ::= & ZY & \\ & Y & ::= & 1Y0 & \mid \epsilon \\ & Z & ::= & 0Z & \mid \epsilon \end{array}$$

would generate the language $L_0 = \{0^a 1^b 0^c \mid b = c \text{ where } a, b, c \in \mathbb{N}\}$*,*
while the grammar

$$\begin{array}{rlcll} \Rightarrow & S & ::= & XZ & \\ & X & ::= & 0X1 & \mid \epsilon \\ & Z & ::= & 0Z & \mid \epsilon \end{array}$$

would generate the language $L_1 = \{0^a 1^b 0^c \mid a = b \text{ where } a, b, c \in \mathbb{N}\}$*.*
G *can generate the union* $L_0 \cup L_1$ *of two languages, i.e.,* $L(G) = \{0^a 1^b 0^c \mid a = b$ ***or*** $b = c \text{ where } a, b, c \in \mathbb{N}\}$*.*

Exercise 2 *Here again $\Sigma = \{0, 1\}$. Design a context-free grammar for the following language:* **[2 marks]**

$$L = \{0^a 1^b 0^c \mid a + b = c \text{ where } a, b, c \in \mathbb{N}\}$$

Solution 2 *The following grammar generates the language L*

$$\begin{array}{rrcll} \Rightarrow & S & ::= & 0S0 & \mid \quad Y \\ & Y & ::= & 1Y0 & \mid \quad \epsilon \end{array}$$