Theories of Computation: Formative Assignment 2

To be handed in on Canvas before **Thursday 17th March, 5pm GMT**

Exercise 1 (Time Complexity in Big-O) *Let us consider two algorithms.*

Algorithm A_1 *has running time* $T_1(n) = \begin{cases} 5n^3 + 2 & for \quad 0 \leq n \leq 3 \\ 7n + 9 & for \quad n \geq 4 \end{cases}$

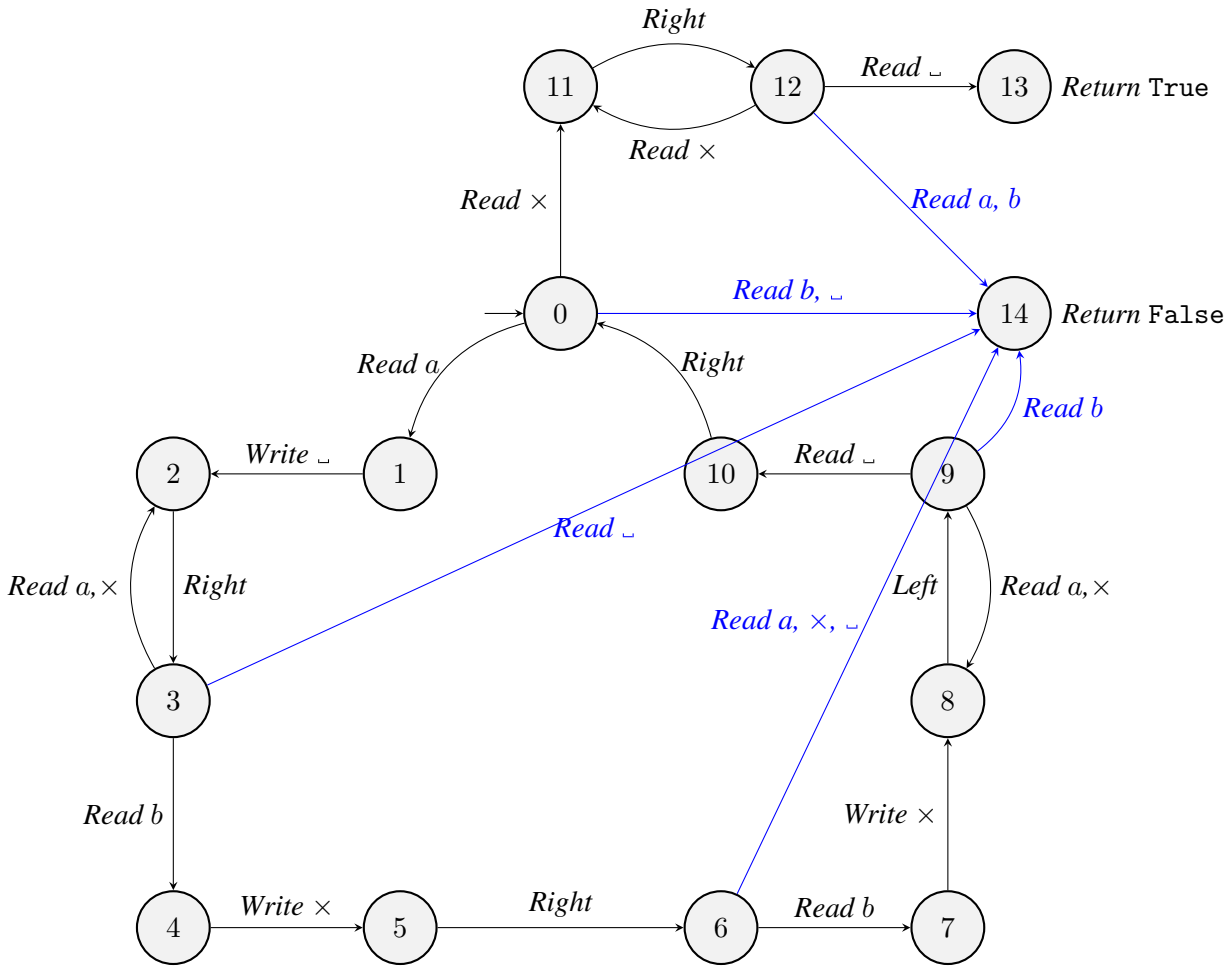
Algorithm A_2 *has running time* $T_2(n) = \begin{cases} 3n^4 + 3 & for \quad 0 \leq n \leq 2 \\ 2n^2 & for \quad n \geq 3 \end{cases}$

1. *Show that* $T_1(n)$ *is* $O(n)$ *and* $T_2(n)$ *is* $O(n^2)$. **[2 marks]**

   Remember: *To justify your claim that* $f(n)$ *is* $O(g(n))$ *provide constants* M *and* C *that satisfy the property that* $\forall n \geq M. f(n) \leq Cg(n)$.

2. *For each* $n \geq 0$ *which algorithm is more efficient? Justify your answer.* **[2 marks]**

Exercise 2 (Turing Machines) *Let us consider a Turing machine* M *on the input alphabet* $\Sigma = \{a, b\}$ *with initial state* 0*, tape alphabet* $T = \{a, b, \sqcup, \times\}$*, return values* $V = \{\texttt{True}, \texttt{False}\}$*, and whose transition function is represented as the following diagram.*



Initially the tape contains a non-empty block of as and bs and is otherwise blank. The head is positioned on the first non blank character.

1. *Give the complete run of the machine* M *above on the word* ab*. At each step, indicate the tape contents, the position of the head, the current state and the instruction (including the result if it is a Read).* **[2 marks]**

***Hint**: No more than 10 steps are needed.*

2. *Without justification, does M accept words abb and $abbb$ (i.e. return* `True` *if given them as input)?* **[2 marks]**

3. *What is the language $\mathcal{L}(M)$ recognised by M?* **[2 marks]**

4. *Use machine M as a macro and design a Turing machine with five states that recognises the language $\mathcal{L} = \{a^{n+1}b^{2n} \mid n \geq 1\}$?* **[2 marks]**