

UNIVERSITY OF BIRMINGHAM

School of Computer Science

First Year Undergraduate

06-35393

35393 LC Theories of Computation

Resit Examinations 2022

[Answer all questions]

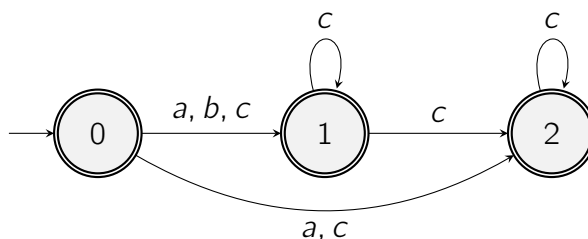
35393 LC Theories of Computation

Answer ALL questions. The paper will be marked out of 60, which will be rescaled to a mark out of 100.

Resit paper

Question 1 : Regular Languages and Automata

Consider the following non-deterministic automaton $\mathcal{A}_0$ on the alphabet $\Sigma = \{a, b, c\}$



(a) Are the following words accepted by $\mathcal{A}_0$? Explain your answer.

- (i) ε
- (ii) abc
- (iii) acc

[6 marks]

(b) Determinize $\mathcal{A}_0$ using the determinization algorithm and then transform the obtained automaton $\mathcal{A}_1$ into a *total* deterministic finite automaton $\mathcal{A}_2$. Is $\mathcal{A}_2$ minimal? Explain your answer.

[9 marks]

Question 2 : Context-free Languages

Consider $\mathcal{L} = \{a^n b^m a^n \mid n \geq 0, m \geq 1\}$, a language on $\Sigma = \{a, b\}$.

(a) Prove that the language $\mathcal{L}$ is not regular. **[7 marks]**

(b) Give a context-free grammar that describes language $\mathcal{L}$. Briefly explain your answer.

[8 marks]

Question 3 : Models of Computation

(a) Are these propositional formulas satisfiable?

(i) $(p \vee q) \wedge (r \vee s) \wedge (\neg p \vee \neg r) \wedge (\neg q \vee \neg s) \wedge (\neg p \vee \neg s)$

(ii) $(p \vee q) \wedge (p \vee \neg q) \wedge \neg p$

Justify your answers.

[6 marks]

(b) If $\mathbf{P} = \mathbf{NP}$, then such satisfiability questions can be decided in polynomial time. Briefly explain why.

[4 marks]

(c) Draw the reduction graph of the following term in λ -calculus with arithmetic:

$$(\lambda x.x + 3)((\lambda y.y + 7)4)$$

[5 marks]

Question 4 : Decidability and Complexity

- (a) Following a spate of malware problems, several companies are reviewing their security procedures.
- (i) The manager of *Very Careful Ltd.* instructs their IT team to write a program that can determine whether an app is able to corrupt data. Can this instruction be carried out? Explain your answer.
 - (ii) The manager of *Somewhat Careful Inc.* instructs their IT team to write a program that can determine whether an app's code contains the signature (the essential part of the code) of a known virus. Can this instruction be carried out? Explain your answer.

[6 marks]

- (b) The following program takes as input a nonempty array p of a's and b's of length $n > 0$.

```
elapse 2 seconds;
for (int i=0; i<n-1; i++){
    elapse 1 second;
    if (p[i]=='b' && p[i+1]=='a') {
        p[i+1] = 'b';
        elapse 1 second;
    }
}
```

For example, if the input is abab, then the output is abbb. All the running time is given by the `elapse` instructions.

Give the best case, worst case, and average case running time for length $n = 3$. For the average case, assume that each entry is independent, with a and b equally likely.

Show that the worse case running time is $O(n)$.

[9 marks]