UNIVERSITY OF BIRMINGHAM

School of Computer Science

Theories of Computation

May 2022

Theories of Computation

Answer ALL questions. The paper will be marked out of 60, which will be rescaled to a mark out of 100.

Exam paper

Question 1 : Regular Languages and Automata

Consider the regular expression $E = (b \mid ab)^*(a \mid \varepsilon)$ on alphabet $\Sigma = \{a, b\}$.

(a) Do the following words match E? Explain your answer.

(i) ε

(ii) $abba$

(iii) aaa

[6 marks]

(b) Give a minimal total DFA that recognizes the language described by E and prove that it is minimal.

[9 marks]

Question 2 : Context-free Languages

Consider the following context-free grammar $\mathcal{G}$ on the alphabet $\Sigma = \{a, b\}$

$$\begin{array}{rlll} \Rightarrow & S & ::= & XX \\ & X & ::= & aXa \mid bXb \mid a \mid b \mid \varepsilon \end{array}$$

(a) Show that the grammar $\mathcal{G}$ is ambiguous. **[7 marks]**

(b) A student is in the process of transforming $\mathcal{G}$ into Chomsky Normal Form and has reached the following:

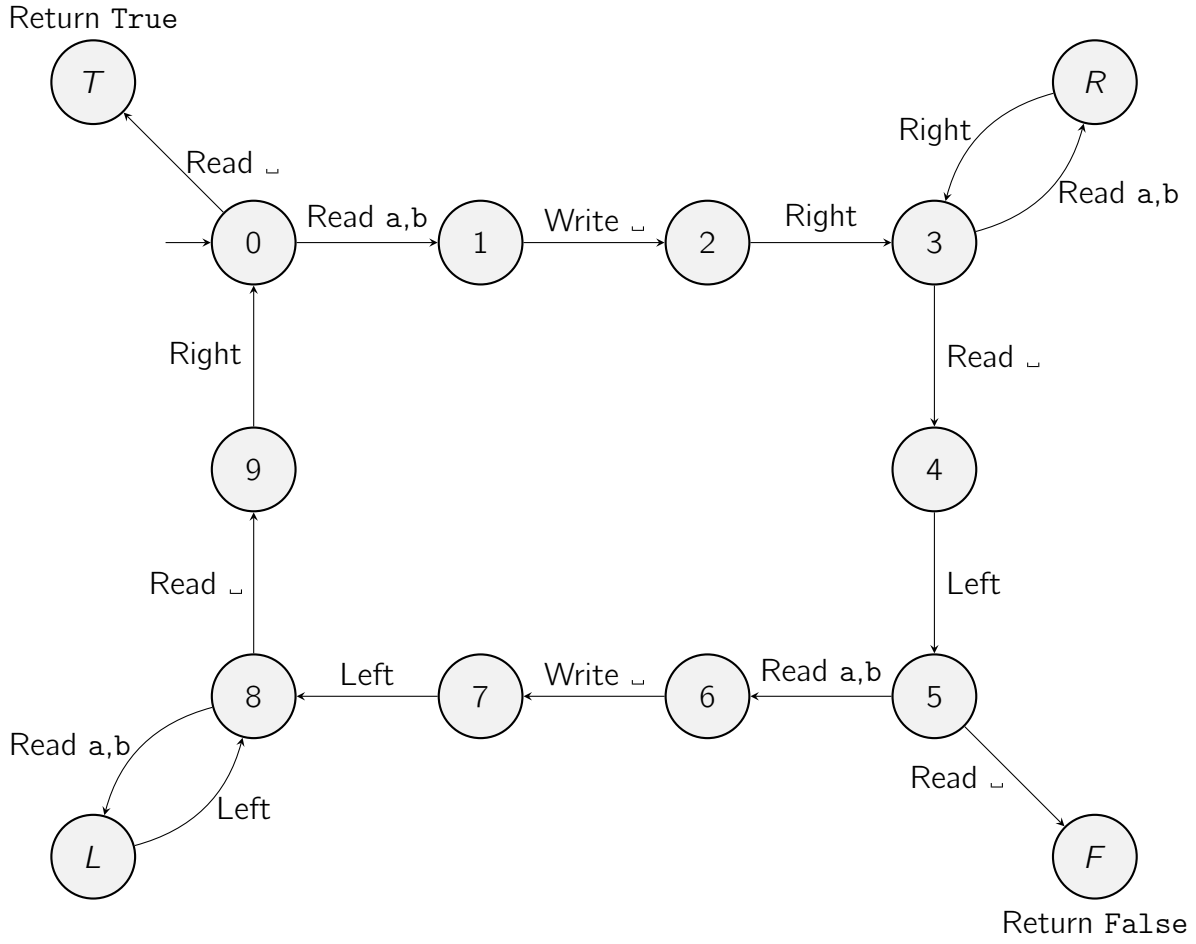
$$\begin{array}{rlll} \Rightarrow & S_0 & ::= & S \\ & S & ::= & XX \\ & X & ::= & AU \mid BV \mid a \mid b \mid \varepsilon \\ & U & ::= & XA \\ & A & ::= & a \\ & V & ::= & XB \\ & B & ::= & b \end{array}$$

The student's next step is to remove the rule $X ::= \varepsilon$. Give the grammar that results from this step.

[8 marks]

Question 3 : Turing Machines and Complexity

Consider the following deterministic Turing machine $\mathcal{M}$ on alphabet $\Omega = \{\mathtt{a}, \mathtt{b}, \sqcup\}$. The tape initially contains a nonempty block of `a`'s and `b`'s on an otherwise blank tape with the head on the leftmost character. The transition function is given by the following diagram:



(a) Trace the behaviour of the machine $\mathcal{M}$ on the word aa. **[7 marks]**

(b) Recall the notation $\sum_{k=0}^{p} x_k$ for $x_0 + x_1 + \cdots + x_p$.

The processing time for a block of length $n > 0$ is as follows.

- In the case where $n = 2p{+}2$ ($p \geq 0$) the number of steps is $(\sum_{k=0}^{p}(8k{+}12)){+}2$.
- In the case where $n = 2p{+}1$ ($p \geq 0$) the number of steps is $(\sum_{k=0}^{p}(8k{+}8)){-}1$.

Show that the complexity of $\mathcal{M}$ is in $O(n^2)$.

[8 marks]

Question 4 : Models of Computation and Decidability

(a) *Removed. Not on the syllabus this year.* **[7 marks]**

(b) A program in Java is said to be *purple* if it either halts or contains (in the body code) an even number of a's. Show that purpleness is undecidable.

[8 marks]