Exercise Sheet
Revisions

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1. **Induction**

   Show that for all $n \in \mathbb{N}$, $5n^3 + n \equiv 0[6]$.

2. **Two's complement on 8 bits**

   In a program, the variables `x`, `y` and `z` represent integers in 8-bit two's complement. These are the values that they currently store:

   - `x` = 0000 1100
   - `y` = 1001 0000
   - `z` = 0111 0111

   (a) Recover the decimal values x, y, $z \in \mathbb{Z}$ that are represented by variables `x`, `y` and `z`.

   (b) Compute the addition of `x` and `z` as binary representations and compare it with the decimal value of $x + y$. Discuss briefly.

3. **Floating points on 8 bits**

   Let us consider an 8-bit floating point encoding with 1 bit for sign s, 3 bits for exponent e, and 4 bits for mantissa m. It therefore represents the number $x = s \cdot m \cdot 2^e$.

   (a) What is the decimal representation of the number represented as
   - 0111 0000
   - 1001 0001

   (b) What is the binary representation of the following numbers as 8-bit floating points
   - $2.5 = 1.25 \times 2^1$
   - $-1.125 = -1.125 \times 2^0$

   (c) Discuss what happens when trying to encode the following numbers as 8-bit floating points
   - $1.03125 = (1 + \frac{1}{32}) \times 2^0$
   - $48 = 1.5 \times 2^5$

4. **Sets and elementhood**

   Let $A = \{x \in \mathbb{R} \mid 0 \leq x \leq 100\}$, $B = \{n \in \mathbb{N} \mid n = p^3 \text{ for some } p \in \mathbb{N}\}$, and $C = \{a, b\}$.

   Give all the elements of the set $(A \cap B) \times C$.

5. **Venn diagrams**

   (a) Draw the Venn diagram for $(A \setminus B) \cup (B \setminus A)$.

Let us abbreviate $(A \setminus B) \cup (B \setminus A)$ as $A \triangledown B$.

(b) Draw the Venn diagram for $(A \triangledown B) \triangledown C$.

6. **Equivalence relation**

   On the set of pairs of real numbers $\mathbb{R}^2$ let us define the relation R as: for all $(x, y), (x', y') \in \mathbb{R}^2$,

   $$((x, y), (x', y')) \in R \iff \text{there exists } a, b > 0 \text{ such that } x' = ax \text{ and } y' = by$$

   Show that R is an equivalence relation.

7. **Order relation**

   On the set of pairs of real numbers $\mathbb{R}^2$ let us define the relation R as: for all $(x, y), (x', y') \in \mathbb{R}^2$,

   $$((x, y), (x', y')) \in R \iff x \leq x' \text{ and } y \leq y'$$

   Show that R is an order relation.

8. **Function and relation**

   Let $A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4\}$ and $R = \{(a, 1), (b, 1), (c, 2), (c, 4), (d, 3)\}$.

   (a) Draw a diagram of the situation with an arrow from an element $x \in A$ on the lefthandside to a an element $y \in B$ on the righthandside when $(x, y) \in R$.

   (b) Is R a function from A to B?

   (c) Take $B' = \{1, 2, 3\}$. Is $R \cap (A \times B')$ a function? If so, is it injective?

9. **Injective/surjective functions**

   Let $f = \begin{array}{c} \mathbb{N}^2 \to \mathbb{N} \\ (n, m) \mapsto n \times m \end{array}$. Is f injective? surjective? bijective?

10. **Countable/uncountable sets**

    For any given $n \in \mathbb{N}$, $n \geq 1$, let $A_n = \{k \in \mathbb{N} \mid 0 \leq k \leq n\}$ and $B_n = \{x \in \mathbb{R} \mid 0 \leq x \leq n\}$

    (a) Is A_n finite, countably infinite or uncountable? Justify your answer.

    (b) Is B_n finite, countably infinite or uncountable? Justify your answer.