UNIVERSITY OF BIRMINGHAM

School of Computer Science

Mathematical and Logical Foundations of Computer Science First Class Test

First Class Test 2020/21

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Question 1 [Set Theory]

(a) (i) What are the elements of GF(2)? **[2 marks]**

(ii) How are addition and multiplication defined for the elements of GF(2)? **[2 marks]**

(iii) List the elements of the set $\{(a, b, c) \in \mathrm{GF}(2)^3 \mid a + b + c = 1\}$. **[4 marks]**

(b) Consider the function $f\colon \mathbb{Z}_5 \to \mathbb{Z}_5$ given by $f(x) = 3x$.

(i) Give the set-theoretic definition of f as a relation from $\mathbb{Z}_5$ to $\mathbb{Z}_5$. **[1 mark]**

(ii) Is f injective, surjective, bijective? Give brief justifications or counterexamples. **[5 marks]**

(c) Discuss the differences between the $\mathbb{R}$ and the Java datatype `float`. **[6 marks]**

Question 2 [Propositional Logic]

(a) Let F be the following proposition: $(R \to \neg P) \to (Q \to R) \to P \to \neg Q$.

(i) Provide an intuitionistic Natural Deduction proof of F. **[3 marks]**

(ii) Provide an intuitionistic Sequent Calculus proof of F. You are allowed to make use of the derived rules seen in the lectures. **[3 marks]**

(iii) Using a truth table, demonstrate whether or not F is (semantically) valid. **[3 marks]**

(iv) Explain how we can prove that F is semantically valid without computing its truth table. **[1 mark]**

(b) Let G be the following proposition: $\neg(\neg P \wedge \neg Q) \to P \vee Q$.

(i) Assume that P and Q are both atomic propositions. Convert G to CNF using the equivalences listed below, and demonstrate whether G is satisfiable. For this question, you are allowed to make use of the equivalences listed on slide 10 of lecture 8, namely:

- $\neg(A \vee B) \leftrightarrow (\neg A \wedge \neg B)$
- $\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$
- $(A \to B) \leftrightarrow (\neg A \vee B)$
- $(A \wedge B) \leftrightarrow (B \wedge A)$
- $(A \vee B) \leftrightarrow (B \vee A)$
- $(\neg\neg A) \leftrightarrow A$
- $((A \wedge B) \wedge C) \leftrightarrow (A \wedge (B \wedge C))$
- $((A \vee B) \vee C) \leftrightarrow (A \vee (B \vee C))$
- $(A \wedge (B \vee C)) \leftrightarrow ((A \wedge B) \vee (A \wedge C))$
- $(A \vee (B \wedge C)) \leftrightarrow ((A \vee B) \wedge (A \vee C))$
- $(A \wedge A) \leftrightarrow A$
- $(A \vee A) \leftrightarrow A$

[3 marks]

(ii) Provide a classical Natural Deduction proof of G. **[3 marks]**

(iii) Provide a classical Sequent Calculus proof of G. Use the version with classical sequents (referred to as version 2 in the lectures). **[3 marks]**

(iv) Explain why one does not need to add Law of Excluded Middle, or Double Negation Elimination rules to the classical version of the Sequent Calculus that uses classical sequents (referred to as version 2 in the lectures). **[1 mark]**