Artificial Intelligence 1

Solutions

Mock Summer Examinations 2024

Note

Answer ALL questions. Each question will be marked out of 20. The paper will be marked out of 60, which will be rescaled to a mark out of 100.

Question 1 Supervised Learning

(a) The following pseudo-code represents one iteration through the training set for gradient descent applied to univariate linear regression.

```
cost = 0;
w0 = 0;
w1 = 0;
For{j=1 to size(trainingSet)}{
      f = w0 + w1*x(j);
      cost = cost + (y(j) - f)^2;
      w0 = w0 - a *(f - y(j));
      w1 = w1 - a *(f - y(j))*x(j);
}
```

Assume that the value of the learning rate, *a* is 1.

Give the numerical values of 'cost', 'w0', and 'w1' at the end of the execution of this pseudo-code for the following training set: $\{(-3, -1), (1, 1), (2, 5)\}$. **Show all your working.** **[10 marks]**

(b) Consider a multivariate data set with 2 classes that are not linearly separable. Is it true that the classes will still be not linearly separable

- (i) if you remove one point from this data?
- (ii) if you remove one feature from this data?

In both cases, justify your answers in the following way: if your answer is yes, then explain why; if your answer is no then give a counter-example. **[10 marks]**

Model answer / LOs / Creativity:

(a) Iteration j=1:
(f=0), cost=1, w0=-1, w1=3,
Iteration j=2:
(f=2), cost=2, w0=-2, w1=2,
Iteration j=3:
(f=2), cost=11, w0=1, w1=8. These are the values at the end of the execution of the pseudo-code.

(b) (i) No: If you remove a point it can become linearly separable. For example, the following 2D labelled data set ((-1,-1), 0), ((1,1), 0), ((-1,1),1), ((1,-1),1) is not linearly separable, but after removing (any) one point from it it becomes linearly separable. (Draw the data for yourself to see.)

(ii) Yes, it remains not linearly separable. That is, a weight vector with which a linear classifier would perfectly label this data does not exist. The reason is a follows: Removing a feature is equivalent to setting the component of the weight vector that multiplies that feature to 0. If this would make the classes linearly separable then this modified weight vector would perfectly label the original data set, which is a contradiction.

Question 2 Clustering

(a) Use hierarchical agglomerative clustering with complete linkage to cluster a 1-dimensional dataset with the following points: 3, 7, 8, 11, 17, 25, 27.

(i) Show your step-by-step calculation of how clusters are formed. If there is a tie, select the first pair from left to right. Draw the resulting dendrogram with heights on one side. Assume that we want to cluster this dataset into 3 clusters. Which 3 clusters would the dendrogram give us? **[10 marks]**

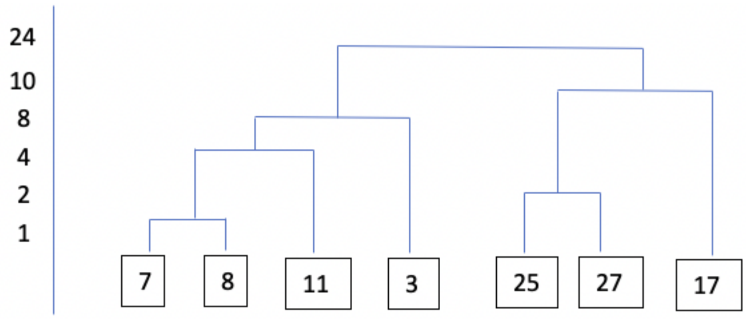
(b) Compute the per-cluster entropy and per-cluster purity of the confusion matrix given below.

Cluster	Entertainment	Financial	Foreign	Metro	National	Sports	Total
#1	1	1	0	11	4	676	693
#2	27	89	333	827	253	33	1562
#3	326	465	8	105	16	29	949
Total	354	555	341	943	273	738	3204

Note: The probability that a member of cluster i belongs to class j is $p_{i,j}$ =number of objects of class j in cluster i/ number of objects in cluster i. Then, entropy of ith cluster is $e_i = -\sum_{j=1}^{L} p_{i,j} \log_2 p_{i,j}$ with L denoting the number of classes. Purity of cluster i is given by $p_i = \max_j p_{i,j}$. **[10 marks]**

Model answer / LOs / Creativity:

(a)
- Combine 7 and 8 with distance 1.
- Combine 25 and 27 with distance 2.
- Combine (7,8) and 11 with distance 4.
- Combine (7,8,11) and 3 with distance 8.
- Combine (25, 27) and 17 with distance 10.
- Combine (3,7,8,11) and (17,25,27) with distance 24.



The dendrogram would give the following three clusters: (3, 7, 8, 11), (25, 27), 17.

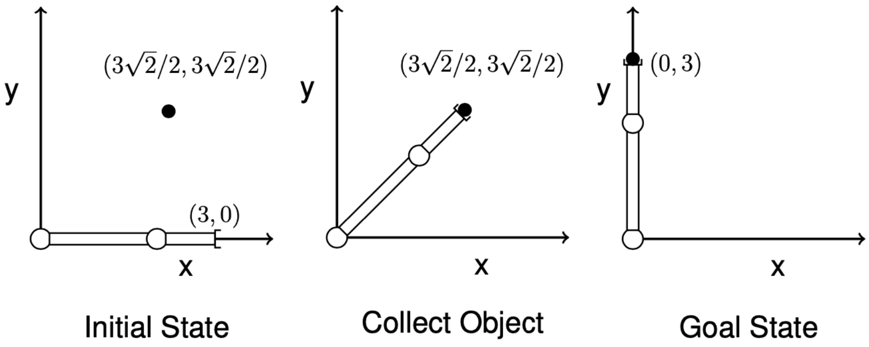
(b) The probabilities can be computed as follows:

$p_{1,1} = 1/693$, $p_{1,2} = 1/693$, $p_{1,3} = 0$, $p_{1,4} = 11/693$, $p_{1,5} = 4/693$, $p_{1,6} = 676/693$.
$p_{2,1} = 27/1562$, $p_{2,2} = 89/1562$, $p_{2,3} = 333/1562$, $p_{2,4} = 827/1562$, $p_{2,5} = 253/1562$, $p_{2,6} = 33/1562$.
$p_{3,1} = 326/949$, $p_{3,2} = 465/949$, $p_{3,3} = 8/949$, $p_{3,4} = 105/949$, $p_{3,5} = 16/949$, $p_{3,6} = 29/949$.

Then, purity of cluster 1 = 676/693, purity of cluster 2 = 827/1562, and purity of cluster 3 = 465/949. Finally, entropy of cluster 1 = 0.2, entropy of cluster 2 = 1.8407, entropy of cluster 3 = 1.6964.

Question 3 Search & Optimisation

A planar robot with two degrees of freedom consists of two links that can rotate around the two rotational joints. The planar robot is placed at the origin as shown in the image below (initial state). The first link has length 2, while the second link has length 1 so that the end effector (i.e., the end of the robotic arm that is used to manipulate objects) is placed at coordinates $(3,0)$ in the initial state.



The goal of the robot is to collect an object placed at coordinates $(3\sqrt{2}/2, 3\sqrt{2}/2)$ and move this object to the position identified by coordinates $(0,3)$, as shown in the Collect Object and Goal State figure above, respectively. This problem can be formulated as a search problem as follows:

- Initial and goal states as shown in the figure above.
- Actions: you can rotate one of the links by 45° or −45°, and you can collect the object only if the end effector is placed above it.
- Nodes are identified by the coordinates of the end effector and by the information if the robot is holding the object. To calculate the coordinates, use the following equations (forward kinematics):

$$x = 2\cos(\theta_1) + \cos(\theta_1 + \theta_2), \quad y = 2\sin(\theta_1) + \sin(\theta_1 + \theta_2),$$

  where θ_1 and θ_2 are the angles of rotation of the first and second joint, respectively, and cos and sin are the cosine and sine functions.
- The cost of each action is equal to 1. Always avoid loopy paths.

To calculate the cosine and sine of a given angle, please refer to the table below.

angle	cosine	sine
0	1	0
45°	$\sqrt{2}/2$	$\sqrt{2}/2$
90°	0	1

(a) Generate the breadth first tree until the goal node is found. **[10 marks]**

When choosing which node to expand in the frontier and all nodes are at the same depth, always expand the node corresponding to the action in the following order: collect object (only if above the object), rotate link 1 by 45°, rotate link 1 by −45°, rotate link 2 by 45° and rotate link 2 by −45°. **Important: we only consider rotations if both coordinates of the position of the end effector are positive.**

Write down the following:

- Search tree produced by breadth-first search, indicating which nodes are in the frontier when the algorithm terminates.
- The solution retrieved by breadth-first search and its cost.
- The order in which the nodes are visited by breadth-first search.

(b) Explain what it means to formulate an optimisation problem as a mathematical definition and describe the components of such a problem. **[10 marks]**

Model answer / LOs / Creativity:

(a) The solution to this part is the following:

- We expand the root node and obtain the following 2 children (the other two are not possible because the coordinates of the end effector would not be positive): $(3\sqrt{2}/2, 3\sqrt{2}/2)$ and $(2+\sqrt{2}/2, \sqrt{2}/2)$.
  We expand node $(3\sqrt{2}/2, 3\sqrt{2}/2)$ and add the following nodes to the frontier: $(3\sqrt{2}/2, 3\sqrt{2}/2)$ (collect the object), $(0, 3)$, $(\sqrt{2}, \sqrt{2}+1)$ and $(\sqrt{2}+1, \sqrt{2})$.
  We expand node $(2+\sqrt{2}/2, \sqrt{2}/2)$ and add the following node to the frontier (since all other nodes would be considered loopy paths):
  $(2, 1)$.
  Finally, we expand node $(3\sqrt{2}/2, 3\sqrt{2}/2)$ (holding the object) and add the following nodes to the frontier:
  $(0, 3)$ (holding the object), $(3, 0)$ (holding the object), $(\sqrt{2}, \sqrt{2}+1)$ (holding the object) and $(\sqrt{2}+1, \sqrt{2})$ (holding the object).
  Since we added the goal node to the frontier, we stop.
- The solution is: rotate link 1 by 45°, collect the object, rotate link 1 by 45°. The cost of this solution is 3.
- The order in which the nodes are visited is the following: $(3, 0)$, $(3\sqrt{2}/2, 3\sqrt{2}/2)$, $(2+\sqrt{2}/2, \sqrt{2}/2)$, $(3\sqrt{2}/2, 3\sqrt{2}/2)$ (holding the object). We do not add the goal node as this node is not visited, but only added to the frontier (stop condition for BFS).

(b) An optimisation problem formulation is a mathematical definition of an optimisation problem. It consists of:

- Design variables. These are variables belonging to pre-defined domains. They represent the candidate solutions of the problem, i.e., possible decisions that need to be made in the problem. There may be one or more design variable in a given optimisation problem.
- Objective functions. These are functions that receive the design variables as input and output a numeric value that the problem aims to minimise or maximise. There may be one or more objective functions in a given optimisation problem.
- Constraints. These are conditions that the design variable(s) must satisfy for the solution to be feasible. They are usually depicted by functions that take the design variable(s) as input and output a numeric value. The constraints then specify the values that these functions are allowed to take for the solution to be feasible. There may be zero or more constraints in a given optimisation problem.