Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers

UNIVERSITY OF BIRMINGHAM

School of Computer Science

Artificial Intelligence 1

Mock Exam 2023

Time allowed: 2 hours

[Answer all questions]

Note

Answer ALL questions. Each question will be marked out of 20. The paper will be marked out of 80, which will be rescaled to a mark out of 100.

Question 1 Clustering

(a) Use hierarchical agglomerative clustering with complete linkage to cluster a 1-dimensional dataset with the following points: 3, 7, 8, 11, 17, 25, 27.

Show your step-by-step calculation of how clusters are formed. If there is a tie, select the first pair from left to right. Draw the resulting dendrogram with heights on one side. Assume that we want to cluster this dataset into 3 clusters. Which 3 clusters would the dendrogram give us? **[10 marks]**

(b) Compute the per-cluster entropy and per-cluster purity of the confusion matrix given below.

Cluster	Entertainment	Financial	Foreign	Metro	National	Sports	Total
#1	1	1	0	11	4	676	693
#2	27	89	333	827	253	33	1562
#3	326	465	8	105	16	29	949
Total	354	555	341	943	273	738	3204

Note: The probability that a member of cluster i belongs to class j is $p_{i,j}$ =number of objects of class j in cluster i/ number of objects in cluster i. Then, entropy of ith cluster is $e_i = -\sum_{j=1}^{L} p_{i,j} \log_2 p_{i,j}$ with L denoting the number of classes. Purity of cluster i is given by $p_i = \max_j p_{i,j}$. **[10 marks]**

Question 2 Supervised Learning

(a) The following pseudo-code represents one iteration through the training set for gradient descent applied to univariate second order polynomial regression.

```
cost = 0;
w0 = 0;
w1 = 0;
w2 = 0;
for j in size(trainingSet)
      f = w0 + w1*x(j) + w2*x(j)^2
      cost = cost + (y(j) - f)^2
      w0 = w0 - a*(f - y(j))
      w1 = w1 - a*(f - y(j))*x(j)
      w2 = w2 - a*(f - y(j))*x(j)^2
endfor
```

Assume that the value of the learning rate, *a* is 1.

Give the numerical values of 'w0', 'w1', 'w2' and 'cost' at the end of this pseudo-code for the following training set: $\{(1, 1), (2, 5)\}$. Show all your working. **[10 marks]**

(b) Consider a multivariate data set with 2 classes that are not linearly separable. Is it true that the classes will still be not linearly separable

- (i) if you remove one point from this data?
- (ii) if you remove one feature from this data?

In both cases, justify your answers in the following way: if your answer is yes, then explain why; if your answer is no then give a counter-example. **[10 marks]**

Question 3 Optimisation

(a) Explain what are constraints in optimisation problems, and how they are usually mathematically depicted in a problem formulation. **[10 marks]**

(b) Consider a regression task represented by a (potentially noisy) training set as follows:

$$D = \{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(n)}, y^{(n)})\},$$

where, for any $i \in \{1, 2, \ldots, n\}$, $x^{(i)}$ and $y^{(i)}$ are real values.

Consider a linear regression model for this regression task. The weights of this model are stored in a 2-dimensional vector $\mathbf{w}$ of real values, and the output of the model for an input x is given by $h(x; \mathbf{w})$. Assume that one proposes to formulate the machine learning problem of learning the weights $\mathbf{w}$ to be adopted for the regression task as an optimisation problem as follows:

$$\text{minimize} \quad f(\mathbf{w}) = \frac{1}{n} \sum_{i=1}^{n} (y^{(i)} - h(x^{(i)}; \mathbf{w}))^2$$

(i) Explain your understanding of what the function $f(\mathbf{w})$ is calculating.

Question 3 continued over the page

(ii) Consider that one wishes to solve the problem given by the proposed formulation using simulated annealing with the design below:

Representation: direct representation of the design variable. In other words, a 2-dimensional vector $\mathbf{w}$ of real values.

Algorithm 1: Initialisation Procedure.

Output: Candidate solution $\mathbf{w}$.

1 $\mathbf{w}$ = new vector of size 2;
2 w_1 = real value picked uniformly at random between 0 (inclusive) and 1 (inclusive);
3 w_2 = real value picked uniformly at random between 0 (inclusive) and 1 (inclusive);
4 **return w**

Algorithm 2: Neighbourhood Operator.

Input: Current solution $\mathbf{w}$.
Output: Neighbour $\mathbf{w}'$.

1 $\mathbf{w}'$ = copy of $\mathbf{w}$;
2 i = value picked uniformly at random from the set $\{1, 2\}$;
3 j = value picked uniformly at random from the set $\{-1, +1\}$;
4 $w'_i = w_i + j$;
5 **return** $\mathbf{w}'$

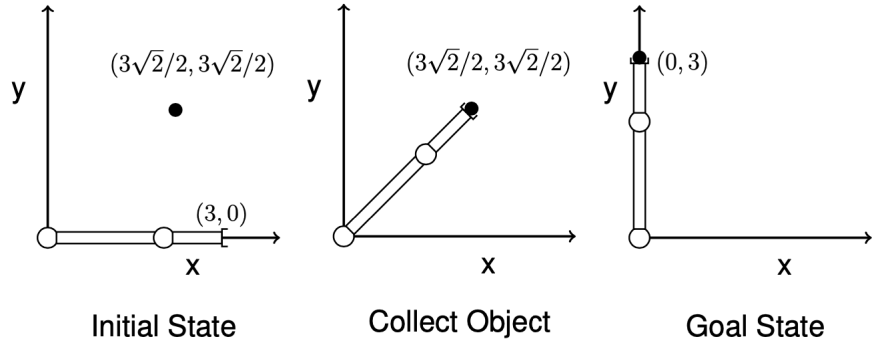
Discuss a key potential weakness of this simulated annealing design in the context of solving the problem given by the proposed formulation.

Note: assume that the proposed formulation truly represents the problem to be solved.

[10 marks]

Question 4 Search

A planar robot with two degrees of freedom consists of two links that can rotate around the two rotational joints. The planar robot is placed at the origin as shown in the image below (initial state). The first link has length 2, while the second link has length 1 so that the end effector (i.e., the end of the robotic arm that is used to manipulate objects) is placed at coordinates $(3, 0)$ in the initial state.



The goal of the robot is to collect an object placed at coordinates $(3\sqrt{2}/2, 3\sqrt{2}/2)$ and move this object to the position identified by coordinates $(0, 3)$, as shown in the Collect Object and Goal State figure above, respectively. This problem can be formulated as a search problem as follows:

Question 1 continued over the page

- Initial and goal states as shown in the figure above.
- Actions: you can rotate one of the links by 45° or −45°, and you can collect the object only if the end effector is placed above it.
- Nodes are identified by the coordinates of the end effector and by the information if the robot is holding the object. To calculate the coordinates, use the following equations (forward kinematics):

$$x = 2\cos(\theta_1) + \cos(\theta_1 + \theta_2), \quad y = 2\sin(\theta_1) + \sin(\theta_1 + \theta_2),$$

  where θ_1 and θ_2 are the angles of rotation of the first and second joint, respectively, and cos and sin are the cosine and sine functions.
- The cost of each action is equal to 1. Always avoid loopy paths.

To calculate the cosine and sine of a given angle, please refer to the table below.

angle	cosine	sine
0	1	0
45°	$\sqrt{2}/2$	$\sqrt{2}/2$
90°	0	1

(a) Consider the state space of the above problem. In a real-life context, the robotic arm can rotate its links by any specified value, even non-integer values, e.g., rotate by 22.5° or 1.6666°. What impact would this have on the execution of a breadth first search on this problem and, more generally, how would this relate to the formulation of a search problem? **Justify** your answer. **[10 marks]**

(b) Generate the breadth first tree until the goal node is found. **[10 marks]**

When choosing which node to expand in the frontier and all nodes are at the same depth, always expand the node corresponding to the action in the following order: collect object (only if above the object), rotate link 1 by 45°, rotate link 1 by −45°, rotate link 2 by 45° and rotate link 2 by −45°. **Important: we only consider rotations if both coordinates of the position of the end effector are positive.**

Write down the following:

- Search tree produced by breadth first search, indicating which nodes are in the frontier when the algorithm terminates.
- The solution retrieved by breadth first search and its cost.
- The order in which the nodes are visited by breadth first search.

End of Paper

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.